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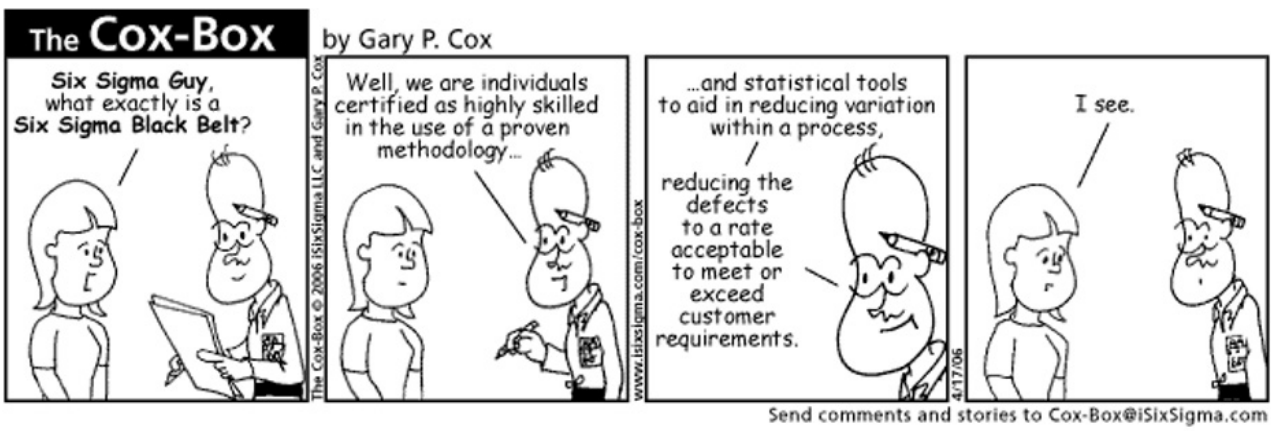
SPC

LESSON: Quality Methods - Introduction to Control Charts

Quality Methods

Control Charts for Variables



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Commonly Used Control Charts

Variable data (characteristics that are measurable on a numerical scale)

- $\overline{X}$ and R-charts
- $\overline{X}$ and s-charts
- Charts for individuals (X-MR or I-MR charts)

Attribute data (count data; qualitative info: does a item conform or not?)

- For “defectives” (p-chart, np-chart)
- For “defects” (c-chart, u-chart)

Charts for Variables vs. Attributes

Variable Charts

- **Quantitative: measurements**
- Track both **mean** and **variation**
- Higher sampling cost in general
- May identify mean shifts before a large number of nonconforming subgroups
- Used at the **lower levels (floor)** as a production tool

Attribute Charts

- **Qualitative: counts**
- Does the sampling unit satisfy taste, color, feel, shape, etc.?
- Must define upfront what a nonconformity is for that process
- Larger sample sizes may be needed to detect nonconformities
- **High level (manager)** performance summary tool

Implementing Control Charts

1. When selecting which variables to measure, choose important **performance parameters** (e.g., process mean, process standard deviation, etc.).
2. Collect trial data for at least k = 25 to 30 subgroups.
3. Construct control charts – is process in control?
   - ◊ Typically, look at variation chart first (R, s, MR)
4. If process is not in control, determine if special cause variation is affecting the out of control points. If so, **eliminate those points and re-plot charts**.
   - ◊ **Do NOT establish control limit**s on an **out of control process!**
5. Implement the charts and plans.

How many data points are needed to set up a control chart?

Shewhart gave the following rule of thumb:

*"It has also been observed that a person would seldom, if ever, be justified in concluding that a state of statistical control of a given repetitive operation or production process has been reached until he had obtained, under presumably the same essential conditions, a sequence of **not less than k = 25 samples of size n = 4** that are in control."*

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Control chart limits for $\overline{X}$ chart calculated from data

$$\mathrm{CL}_{\overline{X}} : E\left(\overline{X}\right) = \overline{\overline{X}} \longleftarrow \text{The mean of the sample means}$$

$$\mathrm{UCL}_{\overline{X}} : E\left(\overline{X}\right) + 3 \cdot \sigma_{\overline{X}} = \overline{\overline{X}} + 3 \cdot \sigma_{\overline{X}}$$

$$\mathrm{LCL}_{\overline{X}} : E\left(\overline{X}\right) - 3 \cdot \sigma_{\overline{X}} = \overline{\overline{X}} - 3 \cdot \underbrace{\sigma_{\overline{X}}}_{\text{The standard deviation of the sample means}}$$

Using An Approximation of to Compute the UCL and LCL for the $\overline{X}$ chart

- We use the range variable R to estimate the standard deviation of $\overline{X}$.
- We want the spread WITHIN a subgroup to estimate σ.

$$\sigma_{\overline{X}} = \frac{\hat{\sigma}_X}{\sqrt{n}} \approx \frac{\overbrace{\frac{\overline{R}}{d_2}}^{\text{Unbiased Estimator of } \sigma_X}}{\sqrt{n}} = \frac{\overline{R}}{d_2\sqrt{n}}$$

$$\mathrm{UCL}_{\overline{X}} : \overline{\overline{X}} + 3 \cdot \sigma_{\overline{X}} = \overline{\overline{X}} + \frac{3\overline{R}}{d_2\sqrt{n}} = \overline{\overline{X}} + A_2\overline{R}$$

Simplifying the calculations of the UCL and LCL for the $\overline{X}$ chart

- The constants d2 and A2 are provided in tables for control charts for various sample sizes n, where

$$A_2 = \frac{3}{d_2\sqrt{n}}$$

- A chart with these constants is provided in the QualityMethods_FormulaReference_2019 document.

Important Note in Computing $\hat{\sigma}$

We do not use the following formula for s to estimate the process standard deviation $\hat{\sigma}$ between subgroups.

$$s = \sqrt{\frac{\sum_{i=1}^{k}(\bar{X}_i - \bar{\bar{X}})^2}{k-1}}$$ where k is the number of subgroups

If the sample means, $\overline{X}_i$, are truly different, then the above formula will ________________ the process standard deviation $\hat{\sigma}$.

Then, the LCL and UCL will be _______________ and it will be harder to detect extreme variations in these means, $\overline{X}_i$.

Control chart limits for R chart calculated from data

$$\mathrm{CL}_R : E(R) = \overline{R} \longleftarrow \text{The mean of the sample ranges}$$

$$\mathrm{UCL}_R = E(R) + 3 \cdot \sigma_R = \overline{R} + 3 \cdot \sigma_R$$

$$\mathrm{LCL}_R = E(R) - 3 \cdot \sigma_R = \overline{R} - 3 \cdot \sigma_R$$

The standard deviation of the sample ranges

For the R chart

We can estimate the R chart control limits also.

$$\sigma_R = d_3 \cdot \frac{\overline{R}}{d_2}$$

$$\mathrm{LCL}_R : \overline{R} - 3d_3 \cdot \frac{\overline{R}}{d_2} = D_3\overline{R}$$

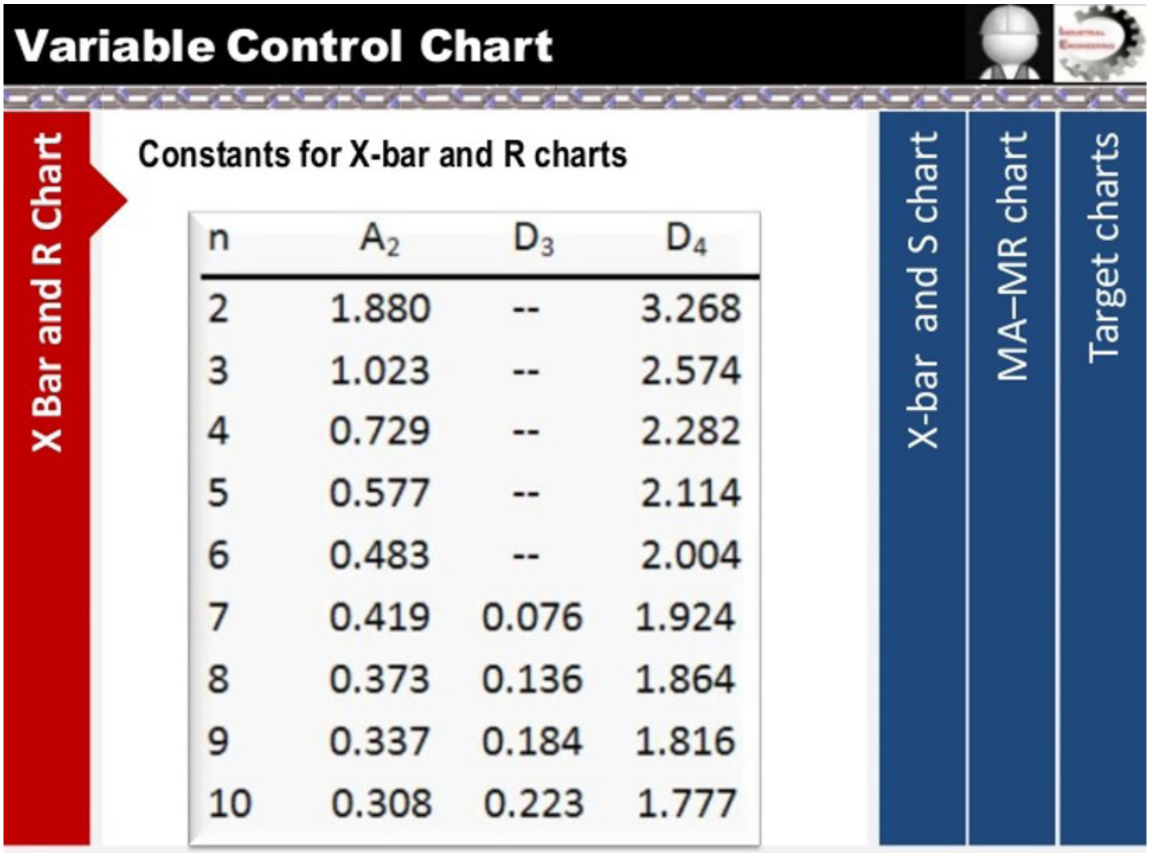
$$\mathrm{UCL}_R : \overline{R} + 3d_3 \cdot \frac{\overline{R}}{d_2} = D_4\overline{R}$$

For the R chart

The constants d_3, d_2, D_3, and D_4 are provided in tables for control charts for various sample sizes *n*, where

$$D_3 = 1 - \frac{3d_3}{d_2} \quad \text{and} \quad D_4 = 1 + \frac{3d_3}{d_2}$$

A chart of these constants is in the QualityMethods_FormulaReference_2019 document.



Constants for X-bar and R charts

n	A_2	D_3	D_4
2	1.880	--	3.268
3	1.023	--	2.574
4	0.729	--	2.282
5	0.577	--	2.114
6	0.483	--	2.004
7	0.419	0.076	1.924
8	0.373	0.136	1.864
9	0.337	0.184	1.816
10	0.308	0.223	1.777

When to use R vs. s charts?

According to many texts and companies:

- If sample size $n < 10$, use Range chart R
- If $n \geq 10$, use Standard Deviation chart s
- Even with use of automated, computer methods for calculation, R charts still popular.

According to most practitioners: **95% of variable control charts use R with n = 4 or 5**.

Control Charts With UCL and LCL Calculated from Data

Once you've determined out-of-control points and can account for the special cause(s) contributing to their existence, **delete these out-of-control points**.

Use the remaining samples to reconfigure the control charts, including the center line and control limits.

Use the remaining samples to determine the estimated mean $\hat{\mu}$ and standard deviation $\hat{\sigma}$ for the process.

Implement the control charts for the continuing process.

Determining the Process Mean and Process Standard Deviation once Process is In Control

The process mean $\hat{\mu}$ is estimated with $\overline{\overline{X}}$ after **removing the out-of-control subgroups**

In order to obtain $\hat{\sigma}$ we must remember how $\hat{\sigma}$ was created for the $\overline{X}$ chart

$$\hat{\sigma} = \frac{\bar{R}}{d_2} \text{ or } \hat{\sigma} = \frac{\bar{s}}{c_4} \text{ or } \hat{\sigma} = \frac{s_p}{c_4}$$

The constants d_2 and c_4 are dependent on the subgroup size n (not the number of subgroups k)

Warning !!

Control Chart Limits are for getting a process in control; most often average measurements are plotted on a control chart

Specification or Tolerance Limits are for determining if individual observations are meeting the customer's demands; individual measurements

$\overline{X}$ and R Control Charts when Parameters or Target Values are Provided

Sometimes we may want to specify (or we are given) target values for the process mean and the process standard deviation. We want to track the process's performance relative to the given target values. Targets are given as: $\overline{X}_0$ or μ_0, and σ_0

An example of this is the bowling example in **last week's assignment** – we were given the **bowling average and standard deviation**; we didn't derive them from the data.

$\overline{X}$ and R Control Chart Limits for Given Targets

- $\bar{X}$ Chart Limits

$$UCL_{\bar{X}} = \overline{X}_0 + 3\sigma_0/\sqrt{n}$$
$$= \overline{X}_0 + A\sigma_0$$

$$LCL_{\bar{X}} = \overline{X}_0 - 3\sigma_0/\sqrt{n}$$
$$= \overline{X}_0 - A\sigma_0$$

- R Chart Limits

$$UCL_R = \overline{R} + 3\sigma_R$$
$$= d_2\sigma_0 + 3d_3\sigma_0$$
$$= D_2\sigma_0$$

$$LCL_R = \overline{R} - 3\sigma_R$$
$$= d_2\sigma_0 - 3d_3\sigma_0$$
$$= D_1\sigma_0$$

Control Chart Example with Targets Given

Example. A process is being set up to produce **10.00 ohm resistors**. The manufacturer has production history which indicates that the resistance is **normally distributed** with a standard deviation of σ_0 **= 0.05 ohm**.

To monitor the process, a sample of **n = 5** resistors will be checked every 15 minutes.

Calculate the CL, UCL, and LCL for the $\overline{X}$ and R charts.

Note: We want to calculate the control chart limits when μ_0 and σ_0 are given in advance, we are not trying to determine the process mean and standard deviation through past or current data

Center line for $\overline{X}$ chart? 10

Control limits for $\overline{X}$ chart?

$$\textbf{UCL: } 10 + 3 \cdot \left(\frac{0.05}{\sqrt{5}}\right)$$
$$\textbf{LCL: } 10 - 3 \cdot \left(\frac{0.05}{\sqrt{5}}\right)$$

Center line for R chart?

$$\textbf{CL: } d_2 \cdot \sigma_0 = 2.326 \cdot 0.05$$

Control limits of R chart?

$$\textbf{UCL: } D_2 \cdot \sigma_0 = 4.918 \cdot 0.05$$
$$\textbf{LCL: } D_1 \cdot \sigma_0 = 0 \cdot 5 = 0$$